Galois groups of 2-adic fields of degree 12 with automorphism group of order 6 and 12

Chad Awtrey and Christopher R. Shill

Abstract Let p be a prime number and n a positive integer. In recent years, several authors have focused on classifying degree n extensions of the p-adic numbers; the most difficult cases arising when $p \mid n$ and n is composite. Since current research has dealt with $n \leq 10$ when $p = 2$, this paper considers degree 12 extensions of the 2-adic numbers. Focusing on extensions whose automorphism groups have order 6 or 12, we compute the Galois group of each extension (or of the normal closure for non-Galois extensions), and identify the group as a transitive subgroup of S_{12}. Our method for computing Galois groups is of interest, since it does not involve factoring resolvent polynomials (which is the traditional approach).

1 Introduction

The p-adic numbers $\mathbf{Q}_p$ are foundational to much of 20th and 21st century number theory (e.g., number fields, elliptic curves, L-functions, and Galois representations) and are connected to many practical applications in physics and cryptography. Of particular interest to number theorists is the role they play in computational attacks on certain unsolved questions in number theory, such as the Riemann Hypothesis and the Birch and Swinnerton-Dyer conjecture (among others). The task of classifying p-adic fields therefore has merit, since the outcomes of such a pursuit can provide computational support to the afore-mentioned problems as well as other number-theoretic investigations.

Classifying extensions of $\mathbf{Q}_p$ entails gathering explicit data that uniquely determine the extensions, including,

Chad Awtrey
Elon University, Campus Box 2320, Elon, NC 27244 e-mail: cawtrey@elon.edu

Christopher R. Shill
Elon University, Campus Box 9017, Elon, NC 27244 e-mail: cshill@elon.edu

1. the number of nonisomorphic extensions for a given prime p and degree n (necessarily finite [15, p. 54]),
2. defining polynomials for each extension, and
3. the Galois group of the extension's polynomial (a difficult computational problem in general).

When $p \nmid n$ (i.e., tamely ramified extensions) or when $p = n$, then items (1)–(3) are well understood (cf. [1, 12]). When $p \mid n$ and n is composite, the situation is more complicated.

In this paper, we study Items (1)–(3) for degree 12 extensions of $\mathbf{Q}_2$, as extensions of smaller degree have already been discussed in the literature ([3, 4, 5, 6, 11, 12, 13]). Specifically, we focus on Galois extensions as well as those extensions whose automorphism groups have order 6. After describing the computation of defining polynomials of such extensions in the next section, we use the final sections of the paper to show that the Galois groups of these polynomials can be computed solely by knowing the Galois groups of their proper subfields. This approach is of interest, since it offers a method for computing Galois groups of local fields that is different from both the resolvent approach [10, 23, 24] and the Newton polygon approach [9, 19].

2 The Number of Extensions and Defining Polynomials

In regards to counting the number of extensions of p-adic fields, some authors have developed what are known as "mass" formulas [14, 18, 21], where the mass of an extension $K/\mathbf{Q}_p$ takes into account the degree of the extension as well as its automorphism group. The mass is defined as:

$$\mathrm{mass}(K/\mathbf{Q}_p) = \frac{[K:\mathbf{Q}_p]}{|\mathrm{Aut}(K/\mathbf{Q}_p)|}.$$

The mass formulas previously mentioned compute the total mass for all extensions of $\mathbf{Q}_p$ of a given degree. As such, different embeddings are counted separately. Therefore these formulas do not give the number of nonisomorphic extensions. Since there is currently no known formula for computing the number of nonisomorphic extensions of $\mathbf{Q}_p$ for a given degree, the approach taken in the literature is to resolve Item (1) by first completing Item (2) (cf. [4, 11, 12, 13]).

The most general reference for the computation of defining polynomials of p-adic fields is [18]. Using the methods of Krasner [14], Pauli-Roblot develop an algorithm for computing extensions of a p-adic field of a given degree by providing a generating set of polynomials to cover all possible extensions. Essential to their method is Panayi's root-finding algorithm [16], which can be used to determine whether two polynomials define isomorphic p-adic fields.

Table 1 shows the number of nonisomorphic extensions of $\mathbf{Q}_p$ of degree n where $p \mid n$ and $n \leq 12$ is composite. This data can be verified by [17], which includes an implementation of the Pauli-Roblot algorithm in its latest release.

Table 1 Number of certain nonisomorphic degree n extensions of $\mathbf{Q}_p$.

(p,n)	(2,4)	(2,6)	(3,6)	(2,8)	(3,9)	(2,10)	(5,10)	(2,12)	(3,12)
#	59	47	75	1834	795	158	258	5493	785

Using the Pauli-Roblot algorithm [18], we see there are 5493 degree 12 extensions of $\mathbf{Q}_2$. Using Panayi's root-finding algorithm to compute the size of each extension's automorphism group, we can show that 27 are Galois extensions and 55 have an automorphism group of order 6. For convenience, Tables 2 and 3 give sample defining polynomials for these two cases, respectively, along with the ramification index, residue degree, and discriminant exponent of the corresponding extension field.

3 Possible Galois Groups

Having computed a defining polynomial for each extension under consideration, we now turn our attention to determining the Galois group of each polynomial.

Given one of the polynomials f in either Table 2 or 3, let K denote the corresponding extension defined by adjoining to $\mathbf{Q}_p$ a root of f. We wish to compute the Galois group G of f, or equivalently the Galois group of the normal closure of K. Since the elements of G act as permutations on the roots of f, once we fix an ordering on the roots, G can be be considered as a subgroup of S_{12}, well-defined up to conjugation (different orderings correspond to conjugates of G). Since the polynomial f is irreducible, G is a transitive subgroup of S_{12}; i.e., there is a single orbit for the action of G on the roots of f (each orbit corresponds to an irreducible factor of f). Therefore G must be a transitive subgroup of S_{12}. Our method for computing Galois groups thus relies on the classification of the 301 transitive subgroups of S_{12} [20].

However, not all of these 301 groups can occur as the Galois group of a degree 12 2-adic field, as we show next.

Definition 1. Let $L/\mathbf{Q}_p$ be a Galois extension with Galois group G. Let v be the discrete valuation on L and let $\mathbf{Z}_L$ denote the corresponding discrete valuation ring. For an integer $i \geq -1$, we define the **i-th ramification group** of G to be the following set

$$G_i = \{\sigma \in G : v(\sigma(x) - x) \geq i+1 \text{ for all } x \in \mathbf{Z}_L\}.$$

The ramification groups define a sequence of decreasing normal sugroups which are eventually trivial and which give structural information about the Galois group of a p-adic field. A proof of the following result can be found in [22, Ch. IV].

Table 2 Polynomials for all degree 12 Galois extensions of $\mathbf{Q}_2$, including ramification index e, residue degree f, and discriminant exponent c.

	Polynomial	**e**	**f**	**c**
1	$x^{12}+x^6+x^4-x+1$	1	12	0
2	$x^{12}-x^{10}-6x^8-x^6+2x^4+7x^2+5$	3	4	8
3	$x^{12}-78x^{10}-1621x^8+460x^6-1977x^4+866x^2+749$	2	6	12
4	$x^{12}-162x^{10}+26423x^8+125508x^6-64481x^4-122498x^2-86071$	2	6	12
5	$x^{12}-16x^{10}+24x^6+64x^4+64$	2	6	18
6	$x^{12}+52x^{10}-28x^8+8x^6+64x^4-32x^2+64$	2	6	18
7	$x^{12}-156x^{10}+9900x^8-61856x^6+33904x^4+27712x^2+47936$	2	6	18
8	$x^{12}-52x^{10}+1100x^8-12000x^6-61072x^4+62144x^2-62144$	2	6	18
9	$x^{12}+12x^{10}+12x^8+8x^6+32x^4-16x^2+16$	6	2	16
10	$x^{12}+x^{10}+6x^8-3x^6+6x^4+x^2-3$	6	2	16
11	$x^{12}-84x^{10}+444x^8+32x^6-272x^4-320x^2+64$	6	2	22
12	$x^{12}-60x^6+52$	6	2	22
13	$x^{12}+2x^{10}+4x^8+4x^6+4x^4+4$	6	2	22
14	$x^{12}-20x^6+20$	6	2	22
15	$x^{12}-4x^{11}-10x^{10}+16x^9-6x^8+16x^7+4x^6-8x^5+16x^4$ $+16x^3+16x^2+8$	4	3	24
16	$x^{12}+28x^{11}-2x^{10}+16x^9+26x^8+8x^7+20x^6-24x^5-8x^4$ $+32x^3+32x^2+32x+24$	4	3	24
17	$x^{12}+32x^{11}-10x^{10}+8x^9-18x^8+32x^7+20x^6+24x^5-24x^4$ $+32x^3+16x^2-24$	4	3	24
18	$x^{12}-4x^{11}+14x^{10}+36x^9-34x^8-32x^7-48x^6-32x^5+36x^4$ $-16x^3-40x^2-48x+56$	4	3	24
19	$x^{12}-2x^{11}+6x^{10}+4x^9+6x^8+12x^7-4x^6-8x^3+16x^2-8$	4	3	18
20	$x^{12}-8x^{10}-28x^8+40x^6-44x^4+48x^2+40$	4	3	33
21	$x^{12}+8x^{10}-12x^8-24x^6+20x^4-16x^2-24$	4	3	33
22	$x^{12}-8x^{10}-28x^8-8x^6+20x^4+16x^2-24$	4	3	33
23	$x^{12}+4x^{10}+10x^8-8x^6+8x^4+32x^2+8$	4	3	33
24	$x^{12}-24x^{10}+52x^8-8x^6+20x^4+16x^2+40$	4	3	33
25	$x^{12}+28x^{10}-6x^8+40x^6-56x^4-32x^2-56$	4	3	33
26	$x^{12}-4x^{10}+26x^8+8x^6-24x^4+32x^2+8$	4	3	33
27	$x^{12}+36x^{10}+42x^8-40x^6+40x^4+32x^2-56$	4	3	33

Lemma 1. *Let $L/\mathbf{Q}_p$ be a Galois extension with Galois group G, and let G_i denote the i-th ramification group. Let $\mathfrak{p}$ denote the unique maximal ideal of $\mathbf{Z}_L$ and U_0 the units in L. For $i \geq 1$, let $U_i = 1+\mathfrak{p}^i$.*

(a) For $i \geq 0$, G_i/G_{i+1} is isomorphic to a subgroup of U_i/U_{i+1}.
(b) The group G_0/G_1 is cyclic and isomorphic to a subgroup of the group of roots of unity in the residue field of L. Its order is prime to p.
(c) The quotients G_i/G_{i+1} for $i \geq 1$ are abelian groups and are direct products of cyclic groups of order p. The group G_1 is a p-group.
(d) The group G_0 is the semi-direct product of a cyclic group of order prime to p with a normal subgroup whose order is a power of p.
(e) The groups G_0 and G are both solvable.

Table 3 Polynomials for all degree 12 extensions of $\mathbf{Q}_2$ that have an automorphism group of order 6, including ramification index e, residue degree f, and discriminant exponent c.

	Polynomial	**e**	**f**	**c**
1	$x^{12}-52x^{10}+20x^8-60x^6-32x^4-16x^2-48$	3	4	8
2	$x^{12}+80x^{10}+81x^8-160x^6-117x^4+80x^2+227$	2	6	12
3	$x^{12}-100x^{10}-59x^8+104x^6+387x^4+444x^2+439$	2	6	12
4	$x^{12}-200x^{10}+7956x^8-7360x^6+6192x^4-2176x^2-4672$	2	6	18
5	$x^{12}-864x^{10}-9916x^8+11008x^6+14512x^4+2560x^2+14528$	2	6	18
6	$x^{12}-108x^{10}-171x^8+344x^6-61x^4+468x^2+359$	6	2	16
7	$x^{12}-30x^{10}-5x^8+19x^4+30x^2+1$	6	2	16
8	$x^{12}-3x^{10}+4x^8-3x^6+4x^4+x^2+3$	6	2	16
9	$x^{12}+5x^{10}+4x^8+x^6+4x^4+x^2+3$	6	2	16
10	$x^{12}-12x^{10}+x^8+12x^6+15x^4+16x^2+15$	6	2	16
11	$x^{12}+7x^{10}+4x^8+3x^6-4x^4-x^2-5$	6	2	16
12	$x^{12}+20x^{10}-44x^8-4x^6-16x^4-48$	6	2	16
13	$x^{12}+4x^{10}+x^8+4x^6-x^4+8x^2-1$	6	2	16
14	$x^{12}+10x^6+12$	6	2	22
15	$x^{12}+2x^6+4$	6	2	22
16	$x^{12}-2x^6+4$	6	2	22
17	$x^{12}+14x^6-12$	6	2	22
18	$x^{12}-14x^6-12$	6	2	22
19	$x^{12}+12$	6	2	22
20	$x^{12}+14x^6+12$	6	2	22
21	$x^{12}+8x^6-4$	6	2	22
22	$x^{12}-6x^6-4$	6	2	22
23	$x^{12}-2x^6-4$	6	2	22
24	$x^{12}-4x^{11}+10x^{10}-6x^8-8x^7+12x^6+8x^5+8x^4+16x^2+8$	4	3	24
25	$x^{12}+12x^{11}-4x^{10}+4x^9-12x^8+4x^6-8x^5-4x^4+16x^3+8x^2+16x-8$	4	3	24
26	$x^{12}+12x^{10}-8x^8+12x^6+4x^4-8x^2+8$	4	3	27
27	$x^{12}+16x^{10}+8x^8+4x^6-12x^4-24x^2-24$	4	3	27
28	$x^{12}+32x^{10}+32x^8-4x^6+20x^4+8x^2+24$	4	3	27
29	$x^{12}+16x^{10}+16x^8-4x^6-12x^4+8x^2-8$	4	3	27
30	$x^{12}+8x^{10}+16x^8-4x^6-12x^4+8x^2-8$	4	3	27
31	$x^{12}-20x^{10}+32x^8-12x^6-28x^4-8x^2+24$	4	3	27
32	$x^{12}+4x^{10}-8x^8+12x^6+4x^4-8x^2+8$	4	3	27
33	$x^{12}+20x^{10}-24x^8-4x^6+4x^4-8x^2-24$	4	3	27
34	$x^{12}+6x^{11}+8x^{10}-52x^9-10x^8+24x^7+8x^6+64x^5+28x^4-40x^3-16x^2-16x+40$	4	3	18
35	$x^{12}+12x^{11}+8x^{10}+4x^9+16x^8-12x^7-8x^6+8x^5-12x^4+16x^3-8$	4	3	18
36	$x^{12}+2x^{10}-x^8+2x^6+6x^4-4x^2-5$	4	3	30
37	$x^{12}+2x^{10}-11x^8+20x^6+31x^4-30x^2-5$	4	3	30
38	$x^{12}+2x^{10}-x^8-2x^6+2x^4-1$	4	3	30
39	$x^{12}+10x^{10}-99x^8+68x^6+79x^4+74x^2+67$	4	3	30
40	$x^{12}-62x^{10}+33x^8+948x^6+775x^4+162x^2+951$	4	3	30
41	$x^{12}+1858x^{10}+1509x^8-1436x^6+2047x^4+786x^2+203$	4	3	30
42	$x^{12}+18x^{10}+17x^8-28x^6-57x^4+34x^2+39$	4	3	30
43	$x^{12}-38x^{10}-87x^8+20x^6-41x^4+74x^2+95$	4	3	30
44	$x^{12}+24x^{10}-4x^8-28x^4+32x^2+24$	4	3	33
45	$x^{12}+18x^8-56x^4+40$	4	3	33
46	$x^{12}+8x^{10}+28x^8+24x^6+20x^4-16x^2+24$	4	3	33
47	$x^{12}-12x^{10}+6x^8-24x^6-24x^4+32x^2-8$	4	3	33
48	$x^{12}-8x^{10}-28x^8+4x^4+32x^2+8$	4	3	33
49	$x^{12}+24x^{10}-12x^8+64x^6+4x^4+32x^2-56$	4	3	33
50	$x^{12}-24x^{10}-10x^8-16x^6+8x^4-64x^2+56$	4	3	33
51	$x^{12}-14x^8-24x^4-24$	4	3	33
52	$x^{12}+6x^8+8x^4-8$	4	3	33
53	$x^{12}+8x^{10}-4x^8+48x^6-28x^4-40$	4	3	33
54	$x^{12}+28x^{10}+22x^8+24x^6-24x^4+32x^2-8$	4	3	33
55	$x^{12}+8x^{10}+28x^8-24x^6+20x^4+16x^2+24$	4	3	33

Applying this lemma to our scenario, where the polynomial f is chosen from Table 2 or 3, $K/\mathbf{Q}_2$ is the extension defined by f, and G is the Galois group of f, we see that G is a solvable transitive subgroup of S_{12}; of which there are 265 [20]. Furthermore, G contains a solvable normal subgroup G_0 such that G/G_0 is cyclic of order dividing 12. The group G_0 contains a normal subgroup G_1 such that G_1 is a 2-group (possibly trivial), and G_0/G_1 is cyclic of order dividing $2^{[G:G_0]}-1$. Only 134 subgroups have the correct filtration. Moreover, since the automorphism group of $K/\mathbf{Q}_2$ is isomorphic to the centralizer of G in S_{12}, we need only consider those subgroups of whose centralizer orders are 12 or 6.

Direct computation on the 134 candidates shows that 5 groups with centralizer equal to 12 and 5 groups with centralizer order equal to 6 can occur as the Galois group of f (note: there are 8 transitive subgroups of S_{12} with centralizer order equal to 6, but only 5 have the correct filtration). We identify these groups in the table below using the transitive numbering system first introduced in [7]. We also give an alternative notation (in the second column), which is based on naming system currently implemented in [8].

12T1	C_{12}
12T2	C_6C_2
12T3	D_6
12T4	A_4
12T5	$1/2[3:2]4$
12T14	D_4C_3
12T15	$1/2[3:2]dD(4)$
12T18	$[3^2]E(4)$
12T19	$[3^2]4$
12T42	$C_6 \wr C_2$

4 Computation of Galois Groups

While most methods for the determination of Galois groups rely on the machinery of resolvent polynomials [10, 23, 24], ours does not. Instead, we use the list of the Galois groups of the Galois closures of the proper nontrivial subfields of the extension. We call this list the *subfield content* of f.

Definition 2. Let f be an irreducible monic polynomial defining the extension $K/\mathbf{Q}_2$ with Galois group G. Suppose K has s proper nontrivial subfields up to isomorphism. Suppose these subfields have defining polynomials $f_1,\dots,f_s$. Let d_i denote the degree of f_i and let G_i be the Galois group of f_i over $\mathbf{Q}_2$. Then G_i is a transitive subgroup of S_{d_i}. Let j_i denote the T-number of G_i (as in [8]). The *subfield content* of f is the set

$$\{d_1\mathrm{T}j_1, d_2\mathrm{T}j_2, \dots, d_s\mathrm{T}j_s\},$$

customarily sorted in increasing order, first by d_i, then by j_i.

Example 1. For example, consider the first polynomial in Table 2, which defines the unique unramified degree-12 extension of $\mathbf{Q}_2$. Thus the Galois group G of this polynomial is cyclic of order 12. Since the transitive group notation in [8] lists cyclic groups first, the T-number of G is 12T1. By the fundamental theorem of Galois theory, since G has a unique cyclic subgroup for every divisor of its order, f has unique subfields of degrees 2, 3, 4, and 6. The Galois groups of these subfields are cyclic, and thus the subfield content of f is $\{$2T1, 3T1, 4T1, 6T1$\}$.

Example 2. As another example, consider the 15th polynomial in Table 3, which is $f = x^{12} + 2x^6 + 4$. The stem field of f clearly has subfields defined by the polynomials $x^6 + 2x^3 + 4$ and $x^4 + 2x^2 + 4$. Using [12], we see that the degree 6 polynomial has Galois group 6T5 $= C_3 \wr C_2$ and the degree 4 polynomial has 4T2 $= V_4$ as its Galois group. Since V_4 has three quadratic subfields, we know the subfield content of f must contain the set $\{$2T1, 2T1, 2T1, 4T2, 6T5$\}$. Consulting Table 5, we see that this set must be equal to the subfield content of f, as no other option is possible. Notice this also proves that the Galois group of f is 12T18.

In general, to compute the subfield content of one of our polynomials f, we can make use of the complete lists of quadratic, cubic, quartic, and sextic 2-adic fields determined in [12] (these lists include defining polynomials along with their Galois groups). For each polynomial in these lists, we can use Panayi's p-adic root-finding algorithm [16, 18] to test if the polynomial has a root in the field defined by f. If it does, then this polynomial defines a subfield of the field defined by f. Continuing in this way, it is straightforward to compute the subfield content of f.

We could also compute subfield content by realizing each degree 12 extension as a quadratic extension of a sextic 2-adic field. This approach can reduce the number of times Panayi's root-finding algorithm is used to compute the subfield content. Details of this approach can be found in [2].

The process of employing the subfield content of a polynomial to identify its Galois group is justified by the following result.

Proposition 1. *The subfield content of a polynomial is an invariant of its Galois group (thus it makes sense to speak of the subfield content of a transitive group).*

Proof. Suppose the polynomial f defines an extension L/K of fields, and let G denote the Galois group of f. Let E be the subgroup fixing L/K, arising from the Galois correspondence. The nonisomorphic subfields of L/K correspond to the intermediate subgroups F, up to conjugation, such that $E \leq F \leq G$. Furthermore, if K' is a subfield and F is its corresponding intermediate group, then the Galois group of the normal closure of K' is equal to the permutation representation of G acting on the cosets of F in G. Consequently, every polynomial with Galois group G must have the same subfield content, and this quantity can be determined by a purely group-theoretic computation. □

Therefore, if we know that the Galois group of a polynomial f must be contained in some set S of transitive subgroups, and if the subfield contents for the groups in S are all different, we can uniquely determine the Galois group of f by computing its subfield content and matching it with its appropriate Galois group's subfield content.

In light of this observation, our approach for determining the Galois groups of the polynomials in Tables 2 and 3 involves three steps: (1) compute the subfield content for each of the possible 10 Galois groups mentioned at the end of Section 3; (2) compute the subfield content for each of the 82 polynomials under consideration; (3) match up the polynomial's subfield content with the appropriate Galois group's subfield content to determine the Galois group of the polynomial.

Table 4 shows the subfield content for each transitive group of S_{12} whose centralizer order is 12. The final column gives the row numbers of all polynomials in Table 2 that have the corresponding Galois group. Similarly, Table 5 shows the subfield content for each transitive subgroup of S_{12} whose centralizer order is 6. The final column in this table references row numbers of polynomials in Table 3. In each table, the entries in column **Subfields** was computed with [8].

Table 4 Subfield content for transitive subgroups of S_{12} that have centralizer order 12. The **Polynomials** column references row numbers in Table 2; the corresponding polynomials have the indicated Galois group.

T	Subfields	Polynomials
12T1	2T1, 3T1, 4T1, 6T1	1, 3, 7, 8, 20, 21, 22, 23, 24, 25, 26, 27
12T2	2T1, 2T1, 2T1, 3T1, 4T2, 6T1, 6T1, 6T1	4, 5, 6, 15, 16, 17, 18
12T3	2T1, 2T1, 2T1, 3T2, 4T2, 6T2, 6T3, 6T3	9, 11, 13
12T4	3T1, 4T4, 6T4	19
12T5	2T1, 3T2, 4T1, 6T2	2, 10, 12, 14

Table 5 Subfield content for transitive subgroups of S_{12} that have centralizer order 6. The **Polynomials** column references row numbers in Table 5; the corresponding polynomials have the indicated Galois group.

T	Subfields	Polynomials
12T14	2T1, 3T1, 4T3, 6T1	2, 3, 4, 5, 24, 25, 26, 27, 28, 29, 30, 31, 32 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55
12T15	2T1, 3T2, 4T3, 6T2	6, 11, 19, 21
12T18	2T1, 2T1, 2T1, 4T2, 6T5	7, 15, 16
12T19	2T1, 4T1, 6T5	1, 12, 17, 18
12T42	2T1, 4T3, 6T5	8, 9, 10, 13, 14, 20, 22, 23

As a final note, we can compute subfield content for the remaining 124 transitive subgroups of S_{12} that are possible Galois groups of degree 12 2-adic fields. Except for the unique group with centralizer order equal to 3 and a few groups with centralizer equal to 4, none of these groups can be distinguished solely by their subfield content. A complete description of subfield contents for the remaining 124 transitive groups of S_{12} can be found in [2]. Identifying the Galois groups of the remaining 5411 degree 12 2-adic fields from among these groups requires other methods and is the subject of ongoing research.

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